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N – TOTALLY DISCONNECTEDNESS IN NEUTROSOPHIC BITOPOLOGICAL SPACES

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***Abstract* – In this paper we introduce the N - connectedness and N - totally disconnectedness in neutrosophic bitopological spaces and investigate some of properties.**

***Keywords* – Neutrosophic set (NS), neutrosophic topological spaces (NTS), neutrosophic bitopological spaces (NBTS), neutrosophic connected (NC), neutrosophic totally connected (NTC).**

I. INTRODUCTION

The notion of fuzzy sets is introduced by the L.A.Zadeh [7] in 1965. Fundamental concept is used based on the number of generalizations. C.L. Chang [2] introduces the concept of fuzzy topological spaces with properties some examples in 1968. K. Atanassov [6] in 1986 introduced the concept of notion of (Ifs) intuitionistic fuzzy sets The concept of the notion of intuitionistic fuzzy topological spaces by using the notion of (IFS)and SE is introduced by the Later Coker [3].

The extend the concept of intuitionistic fuzzy sets into neutrosophic sets is introduced by the Florentin Smarandache [12] in 2005. Three independent functions namely indeterminacy, membership and non-membership functions which are classified in neutrosophic set. In 2012, A.A.Salama, S.A. Alblowi [9,10] introduced the neutrosophic topological spaces with some properties and examples. The concept of bitopological spaces was introduced by Kelly [5] as an extension of topological spaces in 1963. In 2019 Taha Yasin Ozturk and Alkan Ozkan [13] introduced neutrosophic bitopological spaces. In 2021 Md. Aman Mahbub [8] introduced connectedness concept in intuitionistic fuzzy topological spaces. Saikh Shahjahan Miah [4] introduced separation axioms on fuzzy neutrosophic bitopological spaces using fuzzy neutrosophic open sets.

In this study, we presented connectedness and totally disconnectedness on neutrosophic bitopological spaces and some theorems using Hausdorff space. Also, we used basic definitions of neutrosophic open set, closed set and basic properties are investigated.

II. PRELIMINARIES

Definition 2.1[12]: Let X be a non-empty set then neutrosophic set A is an object having the form $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle : x\in X\}$ where the functions $T_A : X \to]^-0,1^+[$, $I_A : X \to]^-0,1^+[$, $F_A : X \to]^-0,1^+[$ denote the degree of membership function (namely $T_A(x)$), the degree of indeterminacy function (namely $I_A(x)$), and the degree of non-membership (namely $F_A(x)$) respectively of each element $x\in X$ to the set A and $^+0 \le T_A(x) \le I_A(x) \le F_A(x) \le 1^+$, for each $x \in X$.

Remark 2.2[12]: Every neutrosophic set A on a non-empty set X is obviously NS having the form $A=\{\langle x,T_A(x),I_A(x),1-T_A(x)\rangle : x\in X\}$

A neutrosophic set $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle : x\in X\}$ can be identified to an ordered triple $\langle x,T_A,I_A,F_A\rangle$ in $]^-0,1^+[$ on X.

Definition 2.3[12]: Let X be a non-empty set and the NSs A and B be in the form $A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle : x\in X\}$ & $B=\{\langle x,T_B(x),I_B(x),F_B(x)\rangle : x\in X\}$ on X then

a) $A\subseteq B \Leftrightarrow T_A(x)\le T_B(x), I_A(x)\le I_B(x), F_A(x)\ge F_B(x)$ for all $x\in X$

b) $A^C = \{\langle x, 1-T_A(x), 1-I_A(x), 1-F_A(x)\rangle : x\in X\}$

c) $A\cap B = \{x, \langle \min(T_A,T_B), \min(I_A,I_B), \max(F_A,F_B)\rangle\}$

d) $A\cup B = \{x, \langle \max(T_A,T_B), \max(I_A,I_B), \min(F_A,F_B)\rangle\}$

Definition 2.4[12]: The neutrosophic empty sets (0_N) and neutrosophic universal sets (1_N) on X defined as $0_N = \{x, 0,0,1\}$ and $1 = \{x, 1,1,0\}$

Definition 2.5[10,13]: Let X be a non-empty set is a collection of neutrosophic sets on X which is called a neutrosophic bitopology on X

a) $0_N, 1_N \in \tau$
b) $N_1, N_2 \in \tau \Rightarrow N_1 \cap N_2 \in \tau$
c) $\cup\{N_i : i\in\Delta\}\in\tau$, for all sets $\{N_i : i\in\Delta\} \subseteq \tau$

If two different topologies τ_1 and τ_2 on X then (X,τ_1,τ_2) is called a neutrosophic bitopological spaces. Each number of non – empty τ is neutrosophic open sets and the complement of τ is closed sets on X.

Example 2.6: Let $X=\{r\}$ and $\tau_1 = \{0_N, P, 1_N\}$, $\tau_2 = \{0_N, R, 1_N\}$ where $P=\{r,\langle 0.2,0.5,0.8\rangle\}$, $R=\{r,\langle 0.4,0.5,0.6\rangle\}$. Then (X,τ_1,τ_2) is a neutrosophic bitopological spaces.

Definition 2.7[11]: A neutrosophic set $N=\{r,\langle T_N, I_N, F_N\rangle : r\in X\}$ is called a neutrosophic point if for any element $t\in X, T_N(s)=\alpha, I_N(s)=\beta, F_N(s)=\gamma$ for $r=t$ and $T_N(s)=0, I_N(s)=0, F_N(s)=1$ for $r\neq s$, where $0\le\alpha\le 1, 0\le\beta\le 1, 0\le\gamma\le 1$.

A neutrosophic point $P=\{r,\langle T_P(r), I_P(r), F_P(r)\rangle : r\in X\}$ will be denoted by P_N or P_N^r.

Definition 2.8[11]: Let (X,τ) be neutrosophic topological spaces, N be a neutrosophic set in X and P_N be a neutrosophic point in X. then P_N is said to be belongs to N, denoted by $P_N\in N$ if and only if $P_N\le T_M, P_N\le I_M, P_N\le F_M$.

Definition 2.9[4]: A mapping $h:(X,\tau)\to(Y.\delta)$ from NTS (X,τ) to another NTS $(Y.\delta)$ is said to be irresolute if $h^{-1}(M)$ is neutrosophic open set in X for each neutrosophic set M in Y.

Definition 2.10[4]: A mapping $h:(X,\tau)\rightarrow(Y.\delta)$ is a neutrosophic open mapping if $h(M)$ is neutrosophic open set in Y where M is open set in X.

III. N- TOTALLY DISCONNECTEDNESS IN NEUTROSOPHIC BITOPOLOGICAL SPACES

Definition 3.1: A neutrosophic bitopological spaces (X,τ_1,τ_2) is called neutrosophic disconnected if there exists a neutrosophic open sets S and T (X,τ_1,τ_2), $S\neq 0_N$, $T\neq 0_N$ such that $S\cup T=1_N$ and $S\cap T=0_N$. If X is not neutrosophic disconnected then it is said to be neutrosophic connected.

Example 3.2: Let $X=\{r,s\}$ and $\tau_1=\{0_N,S,1_N\}$, $\tau_2=\{0_N,T,1_N\}$ where $S=\{\langle r,0,1,0\rangle\langle s,1,0,1\rangle\}$, $T=\{\langle r,1,0,1\rangle\langle s,0,1,0\rangle\}$. Then (X,τ_1,τ_2) is a neutrosophic bitopological spaces. Here S and T are neutrosophic open sets in (X,τ_1,τ_2), $S\neq 0_N$, $T\neq 0_N$ and $S\cup T=1_N$, $S\cap T=0_N$. Hence (X,τ_1,τ_2) is neutrosophic disconnected.

Example 3.3: Let $X=\{r,s\}$ and $\tau_1=\{0_N,S,1_N\}$, $\tau_2=\{0_N,T,1_N\}$ where $S=\{\langle r,0.45,0.5,0.55\rangle\langle s,0.55,0.5,0.45\rangle\}$, $T=\{\langle r,0.32,0.5,0.68\rangle\langle s,0.42,0.5,0.58\rangle\}$. Then (X,τ_1,τ_2) is a neutrosophic bitopological spaces. Here S and T are neutrosophic open sets in (X,τ_1,τ_2), $S\neq 0_N$, $T\neq 0_N$ and $S\cup T=S\neq 1_N$, $S\cap T=T\neq 0_N$. Hence (X,τ_1,τ_2) is neutrosophic connected.

Definition 3.4: A mapping $h:(X,\tau_1,\tau_2)\rightarrow(Y,\tau_1,\tau_2)$ from neutrosophic bitopological space (X,τ_1,τ_2) to another neutrosophic bitopological space (Y,τ_1,τ_2) is said to be irresolute if $h^{-1}(M)$ is neutrosophic open set in X for each neutrosophic set M in Y.

Proposition 3.5: Let (X,τ_1,τ_2), (Y,τ_1,τ_2) be two neutrosophic bitopological spaces and $h:(X,\tau_1,\tau_2)\rightarrow(Y,\tau_1,\tau_2)$ be neutrosophic irresolute surjection. if (X,τ_1,τ_2) is neutrosophic connected then (Y,τ_1,τ_2) is connected.

Proof: Suppose X is neutrosophic connected. Now we show that Y is neutrosophic connected. If possible, suppose that Y is neutrosophic disconnected. Then there exists neutrosophic open sets S, T in Y, $S\neq 0_N$, $T\neq 0_N$ such that $S\cup T=1_N$, $S\cap T=0_N$. Since *h* is neutrosophic irresolute surjection mapping, $P=h^{-1}(S)\neq 0_N$, $R=h^{-1}(T)\neq 0_N$. Which are neutrosophic open sets in X. Also $h^{-1}(S)\cup h^{-1}(T)=h^{-1}(1_N)\Rightarrow h^{-1}(S)\cup h^{-1}(T)=1_N$. i.e $P\cup R=1_N$. Now $h^{-1}(S)\cap h^{-1}(T)=h^{-1}(0_N)\Rightarrow h^{-1}(S)\cap h^{-1}(T)=0_N$. So $P\cap R=0_N$. Thus, X is neutrosophic disconnected. Which is a contradiction to our supposition, X is neutrosophic connected. Hence Y is neutrosophic connected.

Definition 3.6: A neutrosophic bitopological space (X,τ_1,τ_2) is totally disconnected in which every pair of distinct neutrosophic points can be separated by neutrosophic disconnection of X. this means for every pair of distinct neutrosophic points P_N, R_N in X, there exists neutrosophic disconnection $S\cup T=1_N$, $S\cap T=0_N$ such that $P_N\in S, R_N\in T$.

Example 3.7: : Let $X = \{r, s\}$ and $\tau_1 = \{0_N, S, 1_N\}$, $\tau_2 = \{0_N, T, 1_N\}$ where $S = \{\langle r,0,1,0\rangle\langle s,1,0,1\rangle\}$, $T = \{\langle r,1,0,1\rangle\langle s,0,1,0\rangle\}$. Then (X, τ_1, τ_2) is a neutrosophic bitopological spaces. Here S and T are neutrosophic open sets in (X, τ_1, τ_2), $S \neq 0_N$, $T \neq 0_N$ and $S \cup T = 1_N$, $S \cap T = 0_N$. Consider $P_N = \{\langle r.0,0.5,1\rangle\langle s,0.5,0.1\rangle\}$ and $R_N = \{\langle r.0.5,0,1\rangle\langle s,0,0.5.1\rangle\}$ are neutrosophic distinct points in X. Here $P_N \in S, R_N \notin S$ and $R_N \in T, P_N \notin T$. So for distinct neutrosophic points P_N, R_N in X there exists neutrosophic disconnection $S \cup T = 1_N$, $S \cap T = 0_N$ such that $P_N \in S, R_N \in T$. Hence (X, τ_1, τ_2) is neutrosophic totally disconnected.

Definition 3.8: A neutrosophic connected space which is not properly contained in any larger connected space is called neutrosophic component of neutrosophic connected space.

Proposition 3.9: The components of a neutrosophic totally disconnected space are its neutrosophic points.

Proof: Let X be neutrosophic totally disconnected space. i.e for every pair of distinct neutrosophic points can be separated by a neutrosophic disconnection of X. Now we have to show that every neutrosophic subspace Y of X which contains more than one neutrosophic point is neutrosophic disconnected. If possible suppose P_N, R_N are distinct neutrosophic points in Y. Then $S \cup T = 1_N$, $S \cap T = 0_N$ as X is neutrosophic disconnected space. Now $1_N = (Y \cap 1_N) = Y \cap (S \cup T) = (Y \cap S) \cup (Y \cap T)$. Since $S \cap T = 0_N$ $\Rightarrow (Y \cap S) \cap (Y \cap T) = 0_N$. so Y is neutrosophic disconnected subspace of X. this is a contradiction. Hence there is only one neutrosophic point in Y.

Theorem 3.10: Let X be a Hausdorff space. If X has a neutrosophic open base whose sets are also neutrosophic closed sets the X is neutrosophic totally disconnected.

Proof: Let P_N, R_N be distinct neutrosophic points in X. Since X is Hausdorff space, there exists neutrosophic neighbour hood S in X which is also neutrosophic closed set in X such that $P_N \in S \subseteq$ E. Now $(S \cup S^C) = 1_N, (S \cap S^C) = 0_N$ where $P_N \in S, R_N \in S^C$ and $P_N \neq R_N$. Hence X is neutrosophic totally disconnected space.

IV.CONCLUSION

In this paper we introduced neutrosophic connected in neutrosophic bitopological spaces and some of properties are investigated. Also we introduced neutrosophic totally disconnected space using Harsdorf space in neutrosophic bitopological spaces and given examples with some of properties are investigated.

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